Summer Assignments
AB Calculus

Part 1: The three explorations are all due Monday, August 3.
Each should take about one hour to complete.
You may drop them off in the main office at school or mail them to:

Calculus Summer Assignment
Mrs. Karen Laffey
Red Bank Regional High School
101 Ridge Road
Little Silver, NJ 07739

Part 2: The driving project is due the first day of school: Thursday, September 3.
This project should take about two hours. Don't come to class without it!

Instructions

- Present your work on separate paper. Be neat and organized!
  If necessary, rewrite your work, just as you would an essay for another class.

- Be expressive. Write in complete sentences and show your math clearly.
  Any reader should be able to follow your thinking and its underlying logic.

- When drawing graphs, use an appropriate window, label what the axes represent, and identify key values and points.

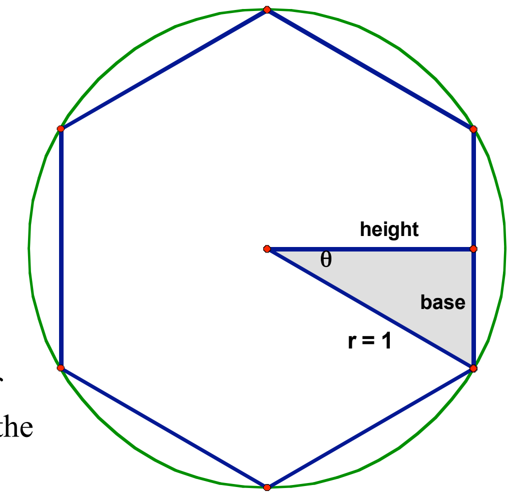
- At the end of each exploration, summarize your findings and conclusions in a well-written paragraph. Answer all the questions posed in the assignment and include any other insights you've made.
  Convince your reader that you thoroughly understand the concepts covered!

Extra Help

Feel free to work with other students on these assignments. It often helps to share ideas.
Also, I will be available via e-mail (KLaffey@rbrhs.org) most of the summer (with the exception of the last week of July.)

Exploration 1: Pi as a Limit

In this exploration, you will use polygons inscribed in a circle of radius $r = 1$ to estimate the value of π. As the polygon's number of sides grows, so should the accuracy of your estimate of π.



Steps

1. Find the area of the circle.

2. First consider a hexagon inscribed in the circle.
Without a protractor, find the measure of angle θ.
Remember, the entire circle covers 2π radians.
(Yes, work in radians, not degrees.)

3. Use trigonometry to find the height, base, and resulting area of the shaded triangle. Don't round off any measurements; keep the sine and cosine expressions.

4. Now find the area of the entire hexagon.
That's your estimate of π. It should be low. Why?

5. Improve your estimate by using a 12-sided polygon instead of a hexagon.
Sketch the new shaded triangle and repeat the previous three steps.
Preserve accuracy. Don't convert to decimals until the final step (4).
The area of the resulting 12-sided polygon should be closer to π. Is it?

6. Repeat five more times, using polygons of successively more sides.
Show your calculations, and present your results in a table: number of sides in column one and resulting areas in column two. Carry all decimals.

7. Generalize your work for an n-sided polygon.
In other words, express the polygon's area as a function of n.
Your expression will involve sine and cosine.
Type your expression into Y= in your calculator. Of course, you'll have to use x, not n.
Be sure your calculator is in radian mode.

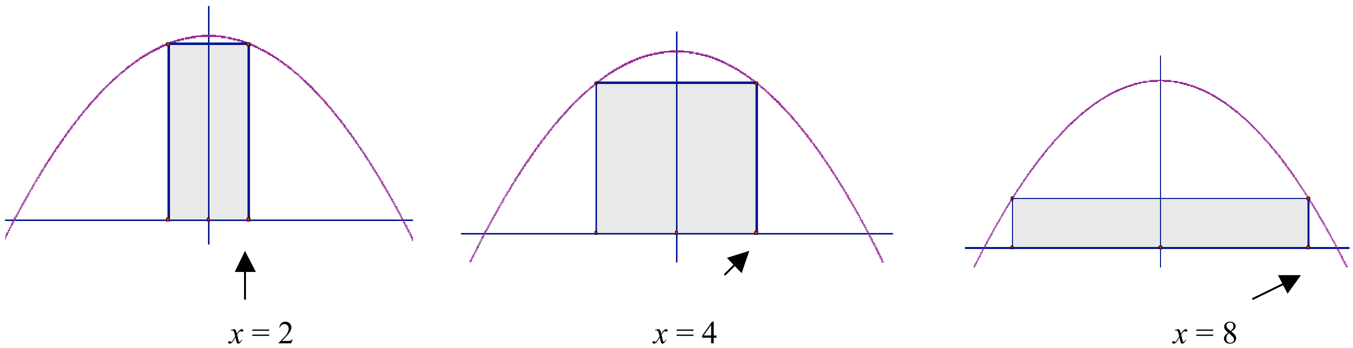
8. Look at your calculator's table, starting with $n = 3$ and incrementing by $\Delta n = 1$.
Do your earlier calculations appear? They should!
As n grows, what value does your expression seem to approach?
Will it ever reach its destination? Look at large values of n to be sure.
How accurate is your estimate of π when n equals 100 or 1000 or 10,000?

9. Look at the graph of your expression and sketch it in your write-up.
A good viewing window is $3 \le x \le 20$ and $0 \le y \le 4$.
Your graph should exhibit a horizontal asymptote. Explain its significance.

<u>Exploration 2</u>: Maximizing Area

In this exploration, you will find the rectangle of largest area that can be inscribed in the parabola $y = 9 - \frac{1}{10}x^2$. Shown below are three such rectangles.

Note that each rectangle is symmetric about the y-axis.
A small x-value (in the lower-right corner) produces a tall, thin rectangle.
A large x-value (in the lower-right corner) produces a short, wide rectangle.



<u>Steps</u>

1. Find the length, height, and resulting area of each of the three rectangles shown. Don't *estimate* each height. Use the parabolic function to find each height *precisely*.

2. Repeat step 1 with several other x-values. Choose x-values that you think will result in even *larger* rectangular areas. Try to converge on the particular x-value that *maximizes* the rectangular area.

3. Present your calculations clearly in a two-column table (x-values in column one and resulting areas in column two) and plot your data on graph paper.

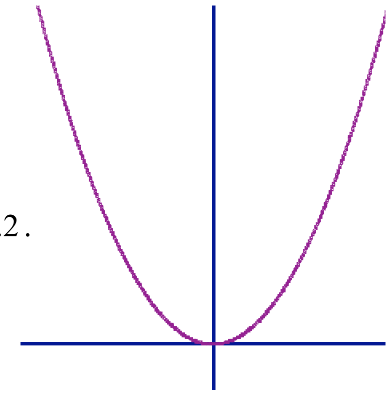
4. Generalize your calculations for *any* x-value. In other words, express the rectangular area as a function of x. Graph that area function on your calculator. Use a window similar to that from your hand-drawn graph.

5. Trace along the area function and also look at its table. Start your table at $x = 0$ and increment by small amounts, such as $\Delta x = .1$ Do your earlier calculations appear? They should!

6. Find the x-value that maximizes the area function. Do this either by tracing along the curve, or by using Calc-Maximum while viewing the graph.

Exploration 3: Slope Patterns on a Curve

In this exploration, you will find the instantaneous slopes at various points along the curve $f(x) = 3x^2$ and look for a pattern to those slopes. Unlike the slopes along a line, the slopes along this curve are continually changing.



Steps

Find the slope at $x = 1$ this way:

1. First, calculate the slope from $x_1 = 1$ to a second point nearby, $x_2 = 1.2$. Use *f* to get precise *y*-values to calculate the slope.
   Then use $m = \frac{y_2 - y_1}{x_2 - x_1}$ to calculate the slope.

2. Improve your result by moving the second point much closer and closer to $x_1 = 1$. For example, let $x_2 =$ 1.02, then 1.002, then 1.0002.

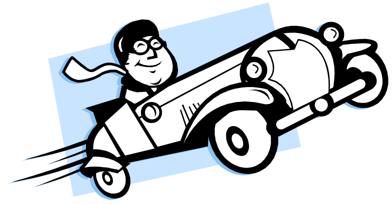
3. You now have five estimates of the instantaneous slope at $x = 1$, each better than the one before. Look at those estimates. Are they approaching a particular value? If so, predict the *true* instantaneous slope at $x = 1$.

4. Repeat steps 1-3 to find the curve's instantaneous slope at $x = 2$, 3, and 4. The instantaneous slopes will grow, since the curve gets steeper as *x* moves east.

5. Now look at your instantaneous slopes for $x = 1$, 2, 3, and 4. Do you see a pattern? You should! Use the pattern to predict the instantaneous slope at $x = 10$ and at $x = -7$. Use the parabola's graph to explain why your predictions make sense.

6. Generalize your findings: find an expression for the slope at *any* *x*-value. That expression is called the *derivative* of $f(x)$ and is denoted as $f'(x)$. It's pronounced "*f* prime of *x*."

7. Now consider again the area function from exploration 2: $A(x) = 18x - \frac{1}{5}x^3$. Its derivative happens to be: $A'(x) = 18 - \frac{3}{5}x^2$. A function's *maximum value* occurs where its *derivative equals zero*. So, solve algebraically the equation $A'(x) = 0$ to find when $A(x)$ is maximized. Remember, after finding *x*, use it to evaluate $A(x)$. Do your results confirm your work in exploration 2? They should!



Driving Project

This project involves taking a 30-minute car ride, collecting data, and analyzing results.
You may work alone or with another calculus student.
If you work with another student, the two of you may submit just one write-up.

Collecting Data

- Record your car's odometer reading *before* starting your car ride.
- With another person* driving, take a 30-minute car ride in a residential area. Find a route that allows you to change speeds throughout the trip.
- During the trip, record your velocity every two minutes ($\frac{1}{30}th$ of an hour). When done, you should have 16 pieces of data for $t = \frac{0}{30}, \frac{1}{30}, \frac{2}{30}, \cdots, \frac{15}{30}$ hours.
- Record the odometer reading *after* your trip.

Analyzing Results

1. Present your data in a table and plot velocity vs. time (in hours) on graph paper. Connect your data points with line segments to form your velocity graph. Clearly identify what each axis represents and include its measurement units. Also identify the increments on each axis.

2. Working from left to right, calculate the area under your velocity graph for each two-minute time interval. You will calculate 15 areas. Most will be trapezoids, but some might be rectangles or triangles. Do you remember the area of a trapezoid?

$$Area = \tfrac{1}{2}(b_1 + b_2)h \qquad \text{(The bases are the parallel sides.)}$$

3. Add your fifteen areas to find the *total* area under your velocity graph.

4. In a few well-written paragraphs, answer these questions:

   - How does the total area under your graph compare to the two odometer readings?

   - What does the area under your velocity graph *represent*? Address units. Divide the total area by ½ hour. What does *that* result represent? Include units.

   - What does the slope during each 2-minute time interval represent? Include units. What does a negative slope imply about your motion?

   - Is your velocity graph a *complete* picture of how your velocity changed during your trip? If not, how could you make it more complete? What other insights can you draw from your results and analysis?

*Another person - a parent would be a good idea!