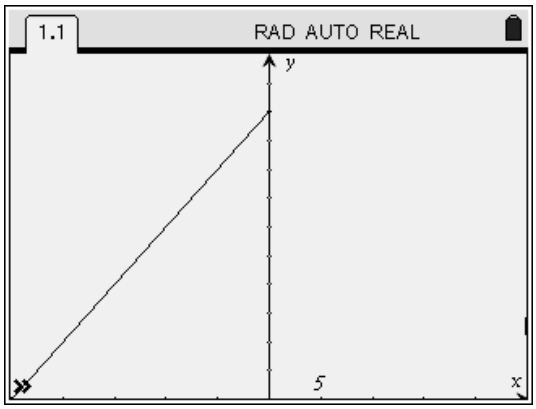
AP© Calculus BC Summer Project

Part I. Welcome to BC Calculus!

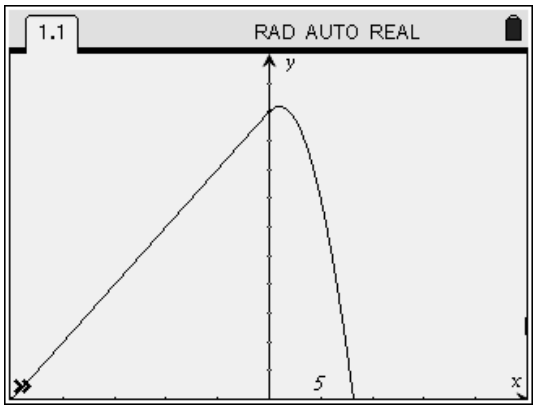
You don't want to forget calculus over the summer, do you? Don't answer that. Anyway, here is a project the will get your juices going and help you remember all about derivatives!

Your job is to design a roller coaster. (90 points)
You will develop a piecewise curve with at least 5 pieces that is differentiable and continuous at all points.

Example: Suppose our roller coaster starts up the "lift ramp" defined by $y = 2x + 50$ when $x \le 0$. The graph of this equation looks like this:



For the next part of the roller coaster, we need to develop a curve which is both continuous and differentiable at $x = 0$. Let's suppose it is a downward parabola in the form: $f(x) = ax^2 + bx + c$. We know that in order to be continuous, $f(0) = 50$, so $c = 50$. In order for $f(x)$ to be differentiable, then $f'(x) = 2ax + b$ must equal the slope of our lift ramp at $x = 0$. So $2ax + b = 2$ when $x = 0$, and b = 2. You can adjust a to any negative value, since we want our parabola to open down. For $a = -1$, $f(x) = -x^2 + 2x + 50$. Our first two sections of our coaster would look like this:



References: www.MasterMathMentor.com and http://shs.westport.k12.ct.us/staples/assignments/Sacilotto/APCALC~1.DOC

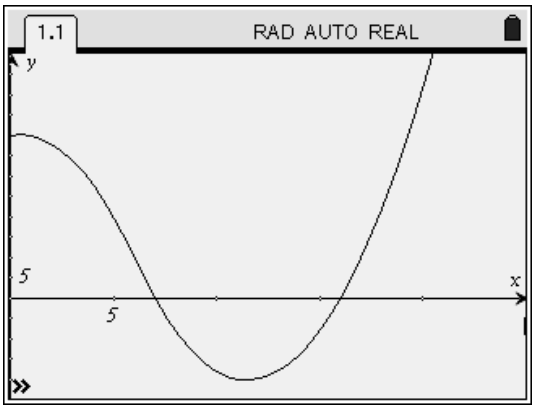
Here is another example: Suppose you have Y2 as $f(x) = -x^2 + x + 40$, $x < 6$, and you wish to connect it with another parabola, but one that opens up. So x = 6 is the point where your new parabola, Y3, would begin.

Define Y3 in the form: $f(x) = a(x-6)^2 + b(x-6) + c$. (It will become apparent soon as to why I would want to do that). Right now, our goal is to find *a*, *b*, and *c*.

For Y2 and Y3 to be continuous at *x* = 6, find Y2(6) = 10 by substitution. Then, Y3(6) = *c*, and *c* = 10.

For Y2 and Y3 to be differentiable at *x* = 6, $Y2'(6) = Y3'(6)$. Find the derivative of Y2: $Y2' = -2x + 1$ and $Y2'(6) = -11$. Then, find the derivative of Y3, $Y3' = 2a(x-6) + b$ and set it equal to -11. Solving for b, gives $b = -11$.

Now, you can pick any positive a, since the parabola will be opening up. For convenience, choose $a = 1$. So your second piece of the coaster would be: $Y3 = 1(x-6)^2 - 11(x-6)) + 10$. Graph this for x > 6.



From the graph, you can see that this goes far below the x-axis. You can play around with the value of a, making it much smaller, until the shape is more reasonable. Remember though that the x-axis really means nothing, since we are not having our curves represent real-life units. The important thing is to have them all graphed on the same scale when you make your final product.

Hopefully you see now why we used the parabola in the form: $f(x) = a(x-6)^2 + b(x-6) + c$. It makes it far easier to work with because when x = 6, many of the terms disappear. So, if your 4th curve was a parabola which needed to start as x =10, your Y4 should be: $Y4 = a(x-10)^2 + b(x-10) + c$. If your Y4 was a cubic, it would be in the form: $Y4 = a(x-10)^3 + b(x-10)^2 + c(x-10) + d$. If it were an exponential, it would be: $Y4 = ae^{k(x-10)}$. Your final equations should be simplified, with like terms combined, and in standard form.

For this project, you will need a piecewise function made up of at least 5 pieces (6 to achieve Expert Level) showing a possible roller coaster. You need to show the graph and a proof that the function you generate is continuous and differentiable at all the points where the curves split into each piece. Round all coefficients to three decimal places. You must use at least three of the following types of functions: linear, trig, exponential, logarithmic, inverse trig, and polynomial. Remember that the end of your last piece must seamlessly connect with the beginning of your first piece.

References: www.MasterMathMentor.com and http://shs.westport.k12.ct.us/staples/assignments/Sacilotto/APCALC~1.DOC

Once you have your function, you will make a model of your roller coaster. It is best to generate the graphs on your calculator. Then:

1) Transfer your graphs to a piece of poster paper. They can be drawn on large graph paper but they must look similar to your calculator-generated graphs. Tape or glue several pieces of poster paper together horizontally. Then, affix the curves to the poster paper.

2) Cut out the roller coaster so that the top of it is your track.

3) Tape or glue one end of the poster paper to the other end. You will end up with a 3-dimensional representation of your roller coaster.

4) Below each of your 5 (or more) curves, write its equation. Make sure your equations are in standard form. Use a heavy dot on the graphs to show transition points. On the poster paper itself, show your proofs that the curves are continuous and differentiable at those points. Round your coefficients to 3 decimal places. (The AP© standard)

5) Name your roller coaster.

Things to remember:

1) Your roller coaster should end at the same height it starts. You may want to have a horizontal line for your loading area. This does **not** count as one of the five curves.

2) Due to physics, a roller coaster never gets as high as its previous high point. Make sure your roller coaster follows the laws of physics!

3) A loop would be spectacular!

This project, as well as Part 2, is due the first day of class in the fall. It will count as a test, and will be graded using the guidelines given in this handout and the attached rubric.

This is not an easy assignment. It will take time. You will present your roller coasters at the beginning of the year and they will be displayed in our classroom. So get an early start and do a good job! Email me at amurray@rbrhs.org if you have problems or questions. Check the website: www.rbrhs.org/wt/murray/ for some great Calculus websites!

References: www.MasterMathMentor.com and http://shs.westport.k12.ct.us/staples/assignments/Sacilotto/APCALC~1.DOC

BC Calculus Summer Coaster Project/PreCalc Review Grading Rubric

Name: _____ Period: _____

	Expert 4	Adequate 3	Reasonable Attempt - 2	Little Attempt 1	Score:
Number of Functions used 5%	>5	5	4 -3	2-1	
Types of Functions Used 5%	>3	3	2	1	
Accuracy of Graphs 30%	>5 are correct	4-5 are correct	2-3 are correct	1 is correct	
Proof that each function is continuous and differentiable at split points 30%	All proofs are complete and correct	One proof is missing or incorrect	Two proofs are missing or incorrect	Three proofs are missing or incorrect	
Construction 15%	The completed coaster is constructed correctly and all information is displayed neatly and clearly on the coaster. It obeys the laws of physics. You include a loop!	One item is missing from the Expert column.	Two items are missing from the Expert column.	Three items are missing from the expert column.	
Creativity 5 %	The completed coaster is eye-catching and colorful. The coaster is named creatively.	Adequate attempt is made to make the coaster attractive.	Reasonable attempt is made to make the coaster attractive.	Little attempt is made to make the coaster attractive.	
PreCalc Review Completion 10%	18-20 correct, including at least one completely correct proof	16-17 correct	11-15 correct	10 or fewer correct	
Late –10% per day					
				Grade:	

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Part 2: Precalculus Review (10 points)

You will be responsible for completing a similar assessment to the one below without a calculator on the first day of class. Prepare yourself!

Complete the following blanks of extremely important concepts. Good luck, you can do it!

List the Pythagorean identities.

1. _____

2. _____

3. _____

List the double angle formulas.

6. $\sin 2x$ = _____

7. $\cos 2x$ = _____

_____

_____

List the sum and difference formulas.

4. $\cos(\alpha \pm \beta)$ = _____

5. $\sin(\alpha \pm \beta)$ = _____

Give the limit definition of e.

8. _____

9. *Expand* : $(x-2y)^6$ _____(No calculator!)

Find the simplest exact value of each of the following.

10. $\sin\frac{7\pi}{6}$ _____

11. $\cos\left(-\frac{\pi}{3}\right)$ _____

12. $\tan\frac{4\pi}{3}$ _____

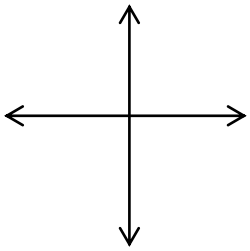
13. $\csc\frac{5\pi}{4}$ _____

14. $\sec\frac{5\pi}{6}$ _____

15. $\cot\frac{2\pi}{3}$ _____

16. For the function below, give the zeros (if none exist write *none*), domain, range, Vertical Asymptotes (VA), Horizontal Asymptotes (HA), and/or points of discontinuity (holes- as ordered pairs) if any exist. Also, sketch its graph.

$f(x)=\frac{x+3}{2x^2+5x-3}$



zeros: _____

domain: _____

range: _____

VA/HA/hole: _____

_____

17. Simplify the expression: $\left(3x^{-1}+x^3\right)^{-2}$ 17. _____

18. Prove: $\cos(A+B)+\cos(A-B)=2\cos A\cos B$ 18. _on separate paper____

19. Prove: $\frac{\sin 2\theta}{1-\cos 2\theta}=\cot\theta$ 19. _on separate paper____

20. Solve for *x*: $\log_5 x+\log_5(x-4)=1$ 20. _____

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