Algebra 2

Summer Assignment 2011

This assignment will be due the first day of school, and a test will be given on this material the second week of school. Brief notes and explanations are included within each section. If additional information is needed, the following websites may be helpful:

http://www.regentsprep.org/Regents/math/ALGEBRA/FormulaSheetAlgebra.pdf

http://regentsprep.org/

http://www.purplemath.com/modules/index.htm

http://www.math.com/students/practice.html

http://www.algebra-class.com

http://www.themathpage.com/alg/algebra.htm

http://www.sosmath.com/algebra/algebra.html

Real Numbers & Number Operations

Define:

Rational Number: ____

Irrational Number: ____

Whole Number:____

Integer: ____

Classify the following numbers as rational, irrational, whole, and/or integer.

1.) $\frac{1}{3}$ 1.) ______________________________

2.) $\sqrt{3}$ 2.) ______________________________

3.) π 3.) ______________________________

4.) -5 4.) ______________________________

5.) 2.1 5.) ______________________________

6.) 0 6.) ______________________________

7.) $1\frac{1}{2}$ 7.) ______________________________

8.) $\sqrt{9}$ 8.) ______________________________

9.) $\frac{14}{3}$ 9.) ______________________________

10.) $-\frac{8}{2}$ 10.) ______________________________

Plot the above 10 numbers on the number line.

Properties of Addition and Multiplication

Property	***Addition***	***Multiplication***
Closure	$a+b$ is a real number	ab is a real number
Commutative	$a+b=b+a$	$ab=ba$
Associative	$(a+b)+c=a+(b+c)$	$(ab)c=a(bc)$
Identity	$a+0=a,\ 0+a=a$	$a\cdot 1=a, 1\cdot a=a$
Inverse	$a+(-a)=0$	$a\cdot\frac{1}{a}=1,\ a\neq 0$
Distributive	$a(b+c)=ab+ac$	

Identify the property as identity, inverse, commutative, associative, or distributive. Identify addition or multiplication.

11.) 5 + (-5) = 0 — 11.) ______________________________

12.) $6\cdot 1=6$ — 12.) ______________________________

13.) $4\cdot(5\cdot 3)=(4\cdot 5)\cdot 3$ — 13.) ______________________________

14.) 4 + 5 = 5 + 4 — 14.) ______________________________

15.) $12\cdot\frac{1}{12}=1$ — 15.) ______________________________

16.) $\alpha+0=\alpha$ — 16.) ______________________________

17.) $\alpha(\beta+\omega)=\alpha\beta+\alpha\omega$ — 17.) ______________________________

18.) (1 + 2) + 3 = 1 + (2 + 3) — 18.) ______________________________

19.) $bc = cb$ — 19.) ______________________________

20.) $1(x) = x$ — 20.) ______________________________

Simplifying and Evaluating Algebraic Expressions

Order of Operations

1.) First, do operations that occur within grouping symbols.

2.) Next, evaluate powers.

3.) Then, do multiplication and divisions from left to right.

4.) Finally, do additions and subtractions from left to right.

Simplify:

21.) $3x-5(x+4)$

22.) $9(x-1)+7(x+4)$

23.) $-(x+4)+6x$

24.) $3x+4x-9-8$

Evaluate:

25.) $(25\div 5+3)^2 \div 16$

26.) $3\cdot 6-4\cdot 5$

27.) $x-xy$ when x = 5, y = -2

28.) $2x+3(x+y)$ when x = 3, y = -1

Solving Linear Equations

Your goal is to isolate the variable on one side of the equation.

For example:

$$\frac{3}{7}x + 9 = 15$$ ***Original equation***

$$\frac{3}{7}x = 6$$ ***Subtract 9 from each side.***

$$x = \frac{7}{3}(6)$$ ***Multiply each side by the reciprocal.***

$$x = 14$$ ***Simplify***

Solve the following equations for x:

29.) $x - 3 = 4x + 15$

30.) $-x + 3 = 7x + 8$

31.) $5(3 - 4x) = 7 - (4 - x)$

32.) $\frac{2}{3}x + 5 = \frac{3}{5}$

33.) $2 - \frac{1}{2}x = 4 - \frac{1}{4}x$

Rewriting Equations and Formulas

Example 1: Rewriting an equation with more than one variable

<u>Solve</u> $7x - 3y = 8$ for y

<u>Steps</u>: $7x - 3y = 8$

$-3y = 8 - 7x$ ***subtract 7x from each side***

$y = -\frac{8}{3} + \frac{7}{3}x$ ***divide each side by -3***

Solve for y:

1. $4x + 8y = 17$

2. $\frac{3}{4}x + 5y = 20$

3. $xy + 2x = 8$

4. $\frac{2}{3}x - \frac{1}{2}y = 12$

Example 2: Rewriting a common formula

Solve $P = 2l + 2w$ for w

Steps: $P = 2l + 2w$

$P - 2l = 2w$ ***subtract 2l from each side***

$\frac{P}{2} - l = w$ ***divide each side by 2***

Solve the formula for the indicated variable:

5. $A = \frac{1}{2}bh$; solve for b

6. $A = \frac{\pi r^2 S}{360}$; solve for S

7. $N = 3a^2 b$; solve for b

8. $A = \frac{1}{2}(b_1 + b_2)h$; solve for h

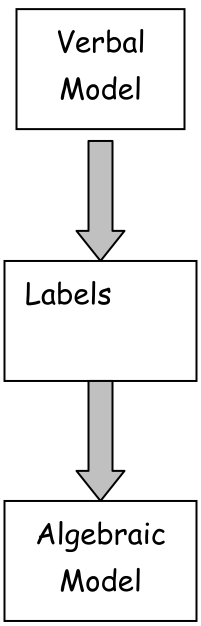
Problem Solving Using Algebraic Models

It is helpful when solving real-life problems to first write an equation in words before you write it in mathematical symbols.

This word equation is called a verbal model.

The verbal model is then used to write a mathematical statement, which is called an algebraic model.

Example 1: The Bullet Train runs between the Japanese cities of Osaka and Fukuoka, a distance of 550 kilometers. When it makes no stops, it takes 2 hours and 15 minutes to make the trip. What is the average speed of the Bullet Train?



DISTANCE = RATE · TIME

Distance = **550 (kilometers)**

Rate = ***r*** **(kilometers per hour)**

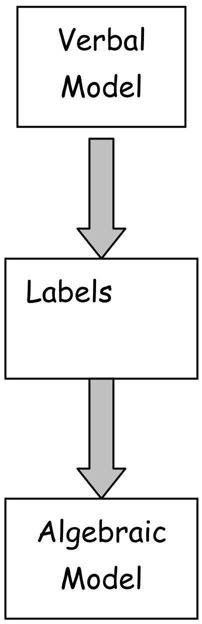
Time = **2.25 (hours)**

$\mathbf{550 = r\,(2.25)}$

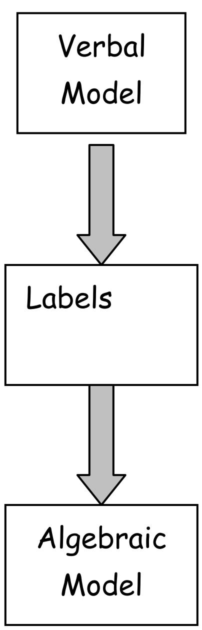
$\frac{550}{2.25} = r \qquad r = 244$

The Bullet Train's Average Speed is about 244 k/hr

<u>Example 2</u>: On August 15, 1995 the Concorde flew 35,035 miles from New York in 31 hours 27 minutes. What was the average speed?



Example 3: A water-saving faucet has a flow rate of at 9.6 cubic inches per second. To test whether you faucet meets this standard, you time how long it takes the faucet to fill a 470 cubic inch pot, obtaining a time of 35 seconds. Find you faucet's flow rate. Does it meet the standard for water conservation?



Solving Linear Inequalities

Solving a linear inequality is a lot like solving a linear equation with **<u>one important exception</u>.** When **<u>multiplying</u>** or **<u>dividing</u>** both sides **<u>BY A NEGATIVE NUMBER</u>**, you must **<u>REVERSE THE INEQUALITY SYMBOL</u>**.

Ex: $-2x < 1$ becomes $x > -\frac{1}{2}$

Ex: $2x < -1$ becomes $x < -\frac{1}{2}$

Ex: $\frac{-x}{2} \geq 1$ becomes $x \leq -2$

Ex: $\frac{x}{2} > -1$ becomes $x > -2$

<u>Compound Linear Inequalities</u>:

Ex: $-2 \leq -2y + 4 \leq 14$

$-6 \leq -2y \leq 10$ …after subtracting 4 from **<u>all three sides</u>**

$3 \geq y \geq -5$ …after dividing **<u>all three sides</u>** by –2 (notice that the symbols reversed)

$-5 \leq y \leq 3$ …after reversing the entire inequality. This last step is not required, but is "cleaner."

This translates to "-5 is less than or equal to *y* <u>which is</u> less than or equal to –1."
This also translates to "-5 is less than or equal to *y* <u>AND</u> *y* is less than or equal to –1."

<u>Solve the inequality and then graph your solution.</u>

1. $3x + 5 < 20$

2. $5 - x < 9$

3. $7x - 3 \geq 13 + 3x$

4. $-y + 5 \leq 3y - 3$

5. $3 < x + 2 < 9$

6. $-2 < 2x + 4 < 10$

7. $x + 3 \leq 5$ or $x - 2 \geq 5$

8. $2x + 1 < -7$ or $3x - 4 > 2$

Linear Equations and Functions

Graphing Functions and Relations

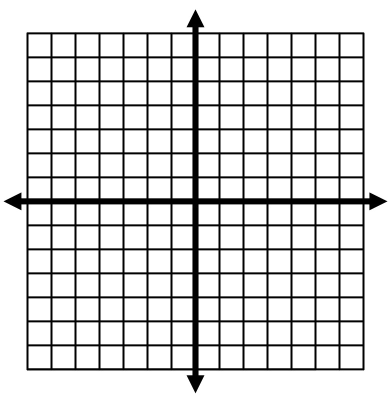
- Recall that for each input or *x* – value, there is exactly one output or *y* – value in order for a *relation* to be classified as a *function*.
- Plot the points (*x*, *y*) listed below on a coordinate grid and use the *vertical line test* to determine whether the relation is a function.

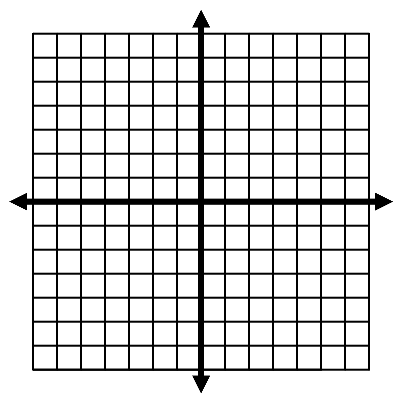
1.

x	-2	-1	0	1	2
y	-1	-1	0	1	1

2.

x	-1	-1	-1	0	1
y	-1	0	1	1	1





- Evaluate the function for the given value of *x*:

3. $f(x) = -x + 3$ when $x = -2$

4. $f(x) = -5 + 8x^2$ when $x = \frac{1}{2}$

5. $f(x) = |x + 3| - 9$ when $x = -4$

6. $f(x) = 2x^3 - 7x^2 + 8$ when $x = -3$

- Recall the formula for the *slope* of a line: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Find the slope of each line going through the given points. Then determine which lines are *parallel* or *perpendicular*.
- Recall that parallel lines have the same slope and perpendicular lines have slope that are opposite reciprocals (ex: $m = \frac{1}{2}$ and $m = -2$)

A. (3, 4) and (1, 6) B. (– 1, 0) and (3, 5) C. (1, 5) and (– 4, – 5)

D. (– 1 – 9) and (2, – 3) E. (– 6, 7) and (– 3, 6) F. (– 1, – 9) and (1, – 3)

- Recall the different forms of a linear function:
  - Slope – intercept: $y = mx + b$, where m= *slope* and b = y – int
  - Standard form: $ax + by = c$, where a, b = coefficients and c = constant
  - Point – slope form: $y - y_1 = m(x - x_1)$, where m = slope and (x_1, y_1) is a point on the line

- Determine the slope and y – intercept of the equation of the line (Hint: rewrite the equation into $y = mx + b$ form).

7. $y = 2x$ $m =$ $b =$

8. $x = -1$ $m =$ $b =$

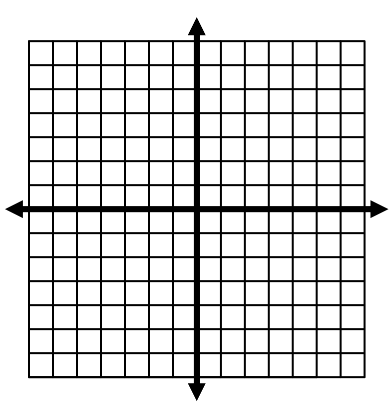
9. $y = 5$ $m =$ $b =$

10. $-2x + y = 10$ $m =$ $b =$

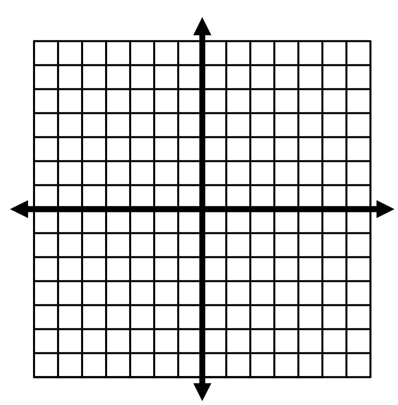
11. $5x - y = 12$ $m =$ $b =$

- Given the slope and y – intercept, draw the line on the graph provided.

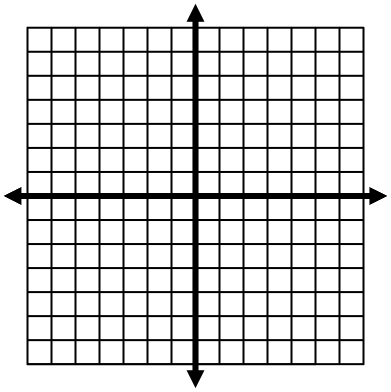
12. $m = 2$, $b = -4$



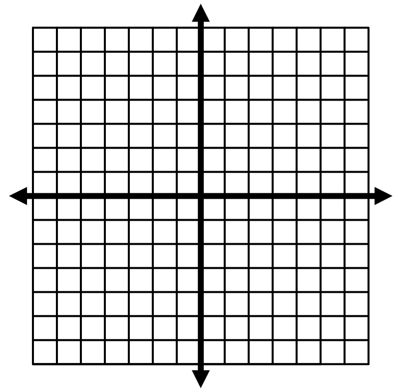
13. $m = 0$, $b = 4$



14. $m = \frac{1}{2}$, $b = 2$

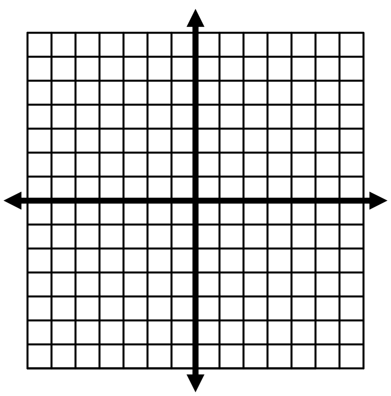


15. $m = -\frac{4}{5}$, $b = -1$

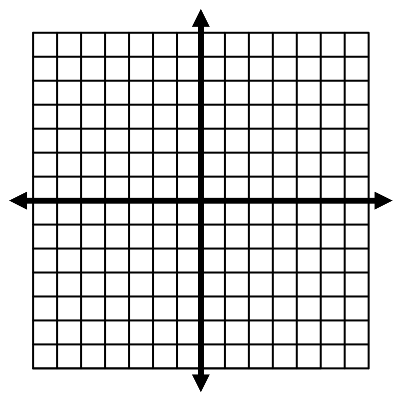


- Graph the equation of the line using any method.

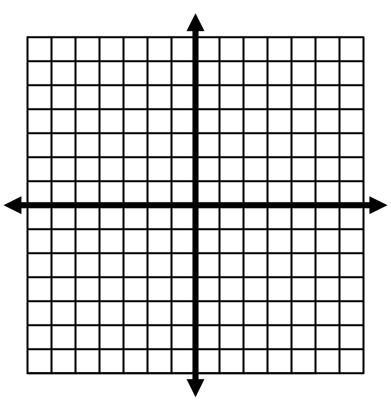
16. $y = 3x$



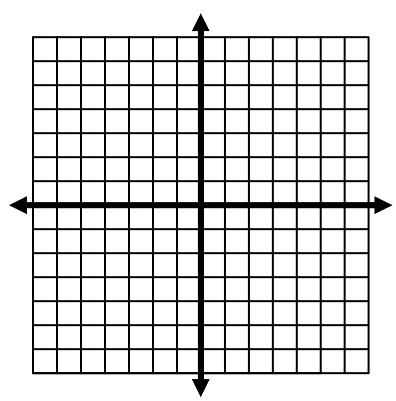
17. $y = -2x - 4$



18. $x + 2y = 1$



19. $x = -5$



- Write the equation of the line given the following:
  - Hint: You may use slope – intercept form ($y = mx + b$) or point – slope form ($y - y_1 = m(x - x_1)$) to write the equation

20. Slope = 2, y – intercept = – 4 20.______________________

21. Slope = 0, y – intercept = 2 21.______________________

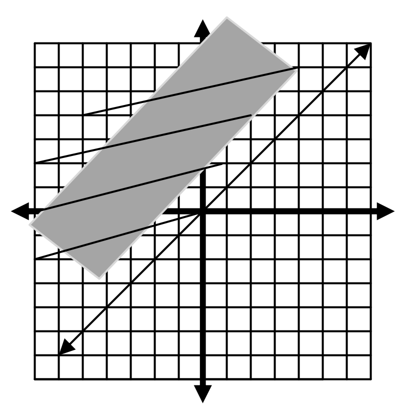
22. Slope = ½, passes through (- 1 , - 1) 22.______________________

23. Slope = 2, passes through (1, - 3) 23.______________________

24. Passes through (1, -6) and (4, -3) 24.______________________

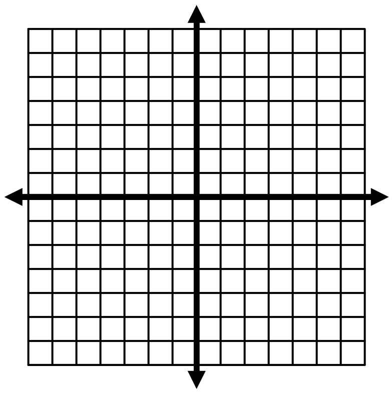
25. Passes through (2, 6) and (-7, 6) 25.______________________

- Graphing Inequalities
  - Recall that > means "greater than", < means "less than", $\geq$ means "greater than or equal to" and $\leq$ means "less than or equal to".
  - Example: Graph the function, $y \geq x$
    - Graph the line using m and b. $m = 1$, $b = 0$
    - Shade the area of the graph that satisfies the inequality
    - Remember to draw a dashed line for > or < signs and a solid line for $\geq$ or $\leq$ signs.

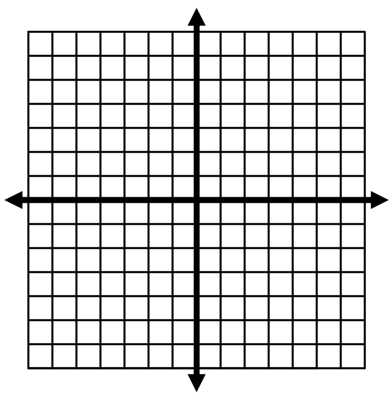


Practice Problems!

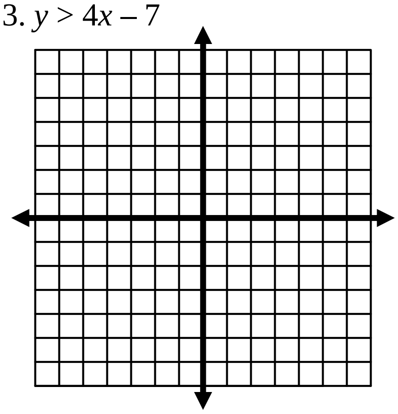
1. $y \geq 2$



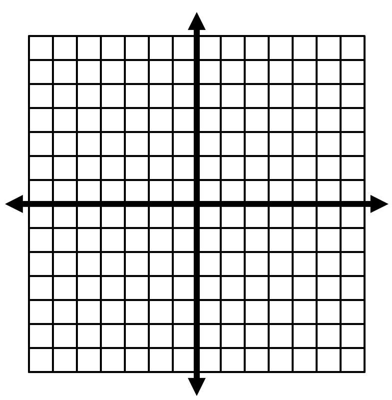
2. $x < -3$



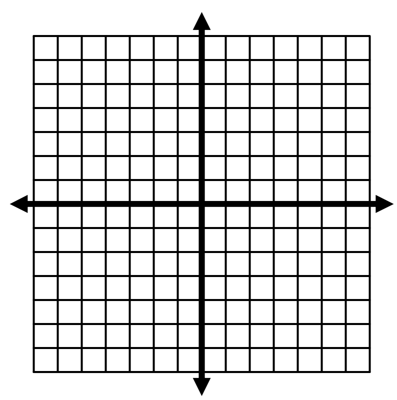
3. $y > 4x - 7$



5. $\frac{1}{2}x + \frac{3}{4}y > 0$



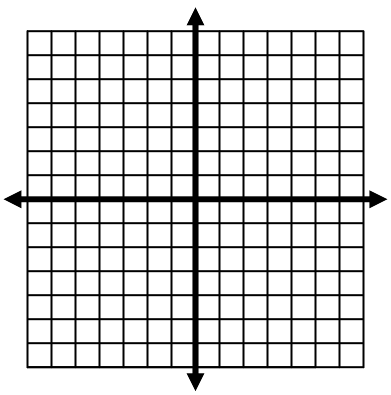
6. $6x + 12y < -24$



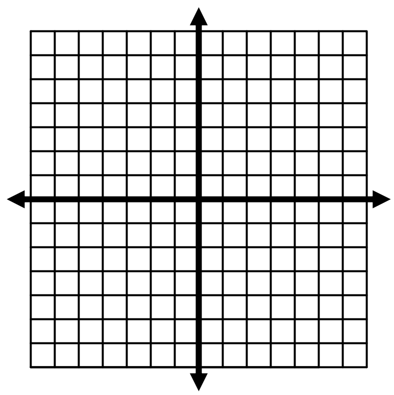
- Graphing Absolute Value Functions
  - Recall the general form of an absolute value function:
  - $y = a|x-h|+k$,
  - Where "*a*" determines how the graph opens (up, down, wide, narrow)
  - Where "*h*" shifts the graph horizontally (ex: $y = |x-3|+2$; 3 units to the right)
  - Where "*k*" shifts the graph vertically (ex: $y = |x-3|+2$; 2 units up)

- Graph the function using a table of values and identify the vertex (point where graph changes directions).

1. $y = |x|-4$



2. $y = |x+3|$



3. $y = 2|x-5|+1$