Section 1.5

Combinations of Functions

Arithmetic Combinations of Functions

Do Now

$f(x) = 2x - 3$

$g(x) = x^2 - 1$

1. $f(x) + g(x)$
2. $f(x) - g(x)$
3. $f(x) * g(x)$
4. $f(x) / g(x)$

Arithmetic Combinations of Functions

Do Now

$f(x) = 2x - 3$

$g(x) = x^2 - 1$

1. $f(x) + g(x) =$

$$= 2x - 3 + x^2 - 1$$

$$= x^2 + 2x - 4$$

Arithmetic Combinations of Functions

Do Now

$f(x) = 2x - 3$

$g(x) = x^2 - 1$

2. $f(x) - g(x) =$

$$= 2x - 3 - (x^2 - 1)$$

$$= 2x - 3 - x^2 + 1$$

$$= -x^2 + 2x - 2$$

Arithmetic Combinations of Functions

Do Now

$f(x) = 2x - 3$

$g(x) = x^2 - 1$

3. $f(x) * g(x) =$

$= (2x - 3)(x^2 - 1)$

$= 2x^3 - 2x - 3x^2 + 3$

$= 2x^3 - 3x^2 - 2x + 3$

Arithmetic Combinations of Functions

Do Now

$f(x) = 2x - 3$

$g(x) = x^2 - 1$

4. $f(x) / g(x) =$

$$= \frac{2x - 3}{x^2 - 1} \qquad x \neq \mp 1 \text{ (or } g(x) \neq 0\text{)}$$

I. Arithmetic Combinations of Functions

Just as two real numbers can be combined with arithmetic operations, two functions can be combined by the operations of addition, subtraction, multiplication, and division to create new functions.

The domain of the arithmetic combinations of functions f and g consists of all real numbers that are common to the domains of f and g. In the case of the quotient $f(x)/g(x)$, there is the further restriction that $g(x) \neq 0$

Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions with overlapping domains. Then, for all x common to both domains, the sum, difference, product, and quotient of f and g are defined as follows:

1. Sum: $(f + g)(x) =$
2. Difference: $(f - g)(x) =$
3. Product: $(fg)(x) =$
4. Quotient: $(f/g)(x) =$

To use a graphing utility to graph the sum of two functions, . . .

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2. Difference: $(f - g)(x) = f(x) - g(x)$
3. Product: $(fg)(x) = f(x) * g(x)$
4. Quotient: $(f/g)(x) = f(x)/g(x)$, $g(x) \neq 0$

To use a graphing utility to graph the sum of two functions, . . .

Sum, Difference, Product, and Quotient of Functions

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2. Difference: $(f - g)(x) = f(x) - g(x)$
3. Product: $(fg)(x) = f(x) * g(x)$
4. Quotient: $(f/g)(x) = f(x)/g(x),\ g(x) \neq 0$

To use a graphing utility to graph the sum of two functions, enter the first function as y_1 and enter the second function as y_2.

Define y_3 as $y_1 + y_2$, then graph y_3

Example 1

Let $f(x) = 7x - 5$ and $g(x) = 3 - 2x$.

Find $(f - g)(4)$

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Let $f(x) = 7x - 5$ and $g(x) = 3 - 2x$.

Find $(f - g)(4)$

$$(f - g)(x) = 7x - 5 - (3 - 2x)$$

$$= 7x - 5 - 3 + 2x$$

$$= 9x - 8$$

$$= 9(4) - 8$$

$$= 36 - 8 = 28$$

II. Compositions of Functions

The **composition** of the function *f* with the function *g*, is

$(f \circ g)(x) = \underline{\ f(g(x))}$

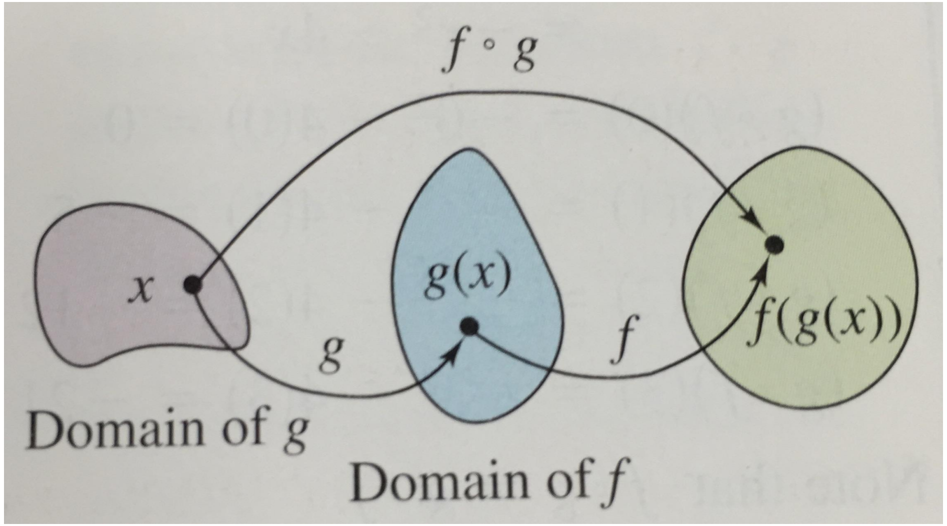
For instance, if $f(x) = x^2$ and $g(x) = x + 1$, the composition of *f* with *g* is

$f(g(x)) = f(g(x)) = f(x + 1) = (x + 1)^2$

This composition is denoted as $f \circ g$ and read as "*f* of *g*."

II. Compositions of Functions

For the composition of the functions *f* with *g*, the domain of $f \circ g$ is <u>the set of all *x* in the domain of *g* such that *g*(x) is in the domain of f.</u>



Example 2

Let $f(x) = 3x + 4$ and let $g(x) = 2x^2 - 1$. Find

(a) $(f \circ g)(x) =$

(b) $(g \circ f)(x) =$

Example 2

Let $f(x) = 3x + 4$ and let $g(x) = 2x^2 - 1$. Find

(a) $(f \circ g)(x) =$

$$= f(2x^2 - 1)$$

$$= 3(2x^2 - 1) + 4$$

$$= 6x^2 - 3 + 4$$

$$= 6x^2 + 1$$

Example 2

Let $f(x) = 3x + 4$ and let $g(x) = 2x^2 - 1$. Find

(b) $(g \circ f)(x) =$

$$= g(3x + 4)$$

$$= 2(3x + 4)^2 - 1$$

$$= 2(9x^2 + 24x + 16) - 1$$

$$= 18x^2 + 48x + 32 - 1$$

$$= 18x^2 + 48x + 31$$

III. Applications of Combinations of Functions

The function $f(\mathrm{x}) = 0.06x$ represents the sales tax owed on a purchase with a price tag of x dollars and the function $g(\mathrm{x}) = 0.75x$ represents the sale price of an item with a price tag of x dollars during a 25% off sale. Using one of the combinations of functions discussed in this section, write the function that represents the sales tax owed on an item with a price tag of x dollars during a 25% off sale.

III. Applications of Combinations of Functions

$f(\text{x}) = 0.06x$

$g(\text{x}) = 0.75x$

$(f \circ g)(\text{x}) = f(0.75x)$

$= 0.06(0.75x)$

$= 0.045x$

So if an item originally cost $100.00, after a 25% off sale, the new item price is $75.00 and the new sales tax will be $4.50.

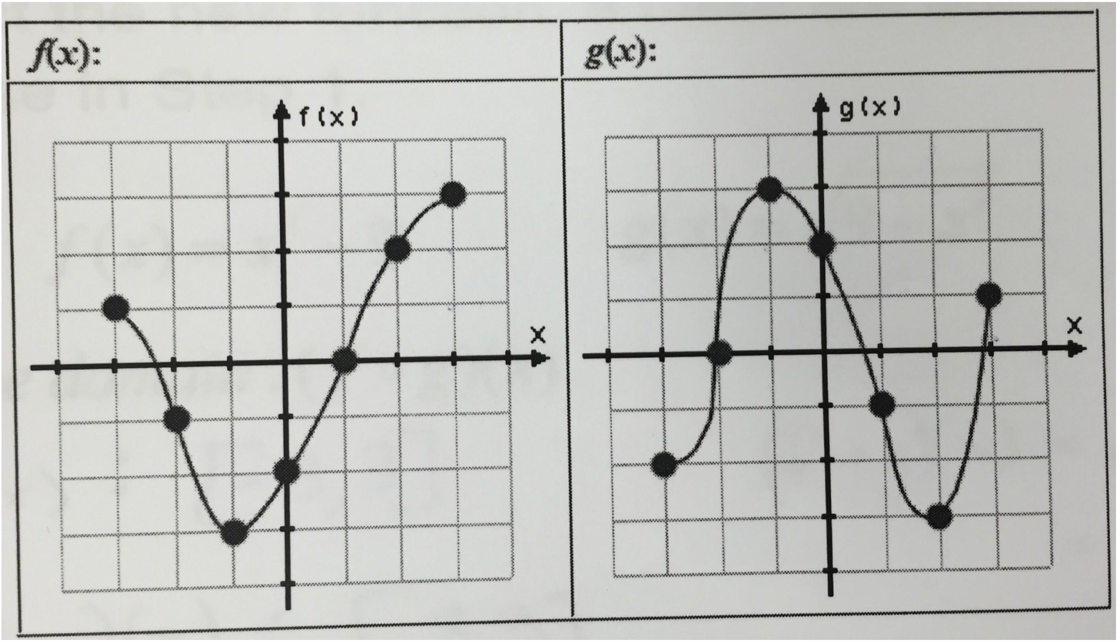
Additional Notes

The composition of f with g is generally not the same as the composition of g with f.

IV. Composition of Functions Graphically

Find $(g \circ f)(3)$

Find $(f \circ g)(-1)$



Practice

Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$

Find $(f + g)(x)$.

Evaluate the sum when $x = 2$.

Practice

Given $f(x) = x^2$ and $g(x) = x - 3$

Find $(fg)(x)$.

Evaluate the sum when $x = 4$.

More Practice

Given $f(x) = x^2 + 2x$ and $g(x) = 2x + 1$, find the following

a. $(f \circ g)(x)$

b. $(f \circ g)(5)$

More Practice

Given $f(x) = x^2 + 2x$ and $g(x) = 2x + 1$, find the following

a. $(g \circ f)(x)$

b. $(g \circ f)(5)$

Finding the Domain of a Composite Function

Given $f(x) = 1/x$ and $g(x) = \sqrt{x}$

State the domain of $(f \circ g)(x)$

(Hint: Find the domain of each first)

Finding the Domain of a Composite Function

Find the domain of the composition $(f \circ g)(x)$ for the functions given by

$f(x) = x^2 - 9$ $g(x) = \sqrt{(9 - x^2)}$